# **Approximation and Optimization Methods**

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# Abstract

Applied Mathematics consists of a wide range of research interests, which characterize the interdisciplinary works of the involved researchers. Our investigations explore the fields of *Numerical Analysis* (numerical modeling, approximation theory, numerical derivation, numerical linear algebra, image analysis for deblurring and motion magnification), *Machine Learning* (explainable Machine Learning, optimization theory, classification problems, transportation problems), *Mechanical Design* (impact attenuators, crash behavior of automotive structures, characterization of automotive composites), *Financial Mathematics* (nonlinear stochastic modeling, temporal graph neural networks, numerical methods for mathematical finance). The shared aim of these works is to provide solutions to complex problems or forecast behaviors. Despite Applied Mathematics research being often driven by specific applications, it offers transversally exploitable solutions and fosters insights into Pure Mathematics, as applied and fundamental knowledge are ultimately inseparable.





**ENVIRONMENT AND ENERGY** 



**Automotive composites.** The research explores the development and optimization of composite materials for automotive applications, focusing on transitioning from fully synthetic reinforcements to hybrid and natural fiber-based solutions. These innovative materials aim to balance performance, sustainability, and cost-effectiveness, making them ideal for energy-absorbing components such as crash boxes. The design process involves the four critical steps in the picture.

**Counterfactual explanations.** Machine Learning algorithms create predictive models that can inherit biases and prejudices of the training data. AI explanations, through mathematical optimization, make these models understandable to humans and highlight potential biases or unfair decisions made by the system. Counterfactuals are intuitive explanations that offer valuable insights into what changes would be needed to alter the outcome of a black-box decision making model, without exposing the underlying algorithm.



Fluid-dynamic models. In the study of realworld problems, the analytical modeling and numerical approximation of the system are fundamental. Models based on Fluid Dynamics can describe complex phenomena, such as a turbulent forced convective fluid flow in a geothermal exchanger arranged in a geothermal field and percolation in a porous medium – for instance, the rainwater infiltration into the soil that allows the prediction of landslide hazard, or the extraction of espresso coffee. These models are approximated with suitable numerical schemes to obtain an approximated solution, as the analytical solution is usually unknown.



## **Research Impact**

The scientific impact of research in Applied Mathematics, due to its collaborative and interdisciplinary nature, lies in the creation and strengthening of long-lasting collaborations among institutions and networks of skilled researchers. The involvement of industries and technological transfer build the economic impact, which means the industrial partners increase their competitiveness worldwide and eventually gain market portions. Last but not least, approximation and optimization fields yield a social impact by targeting the sustainable development goals dealing with environment, energy, responsible consumption and production.

#### **International Collaborations**





### **Financial Supports and Membership**

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